Persistence filtering under non-stationary forcing: a formal core

Meow-Ludo
meow@meow-meow.io

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Abstract

The framework developed in *The First Filter* generates its results by inversion: it treats conserved biological features as preserved products and solves backwards for the regime that made them. Twelve such derivations are stated in the companion *Claims and Derivations* document, each as a chain of implications. This paper supplies the formal core those chains have been running on implicitly. It does three things. First, it distinguishes three inference types that the chains currently render with a single arrow, and marks each of the twelve. Second, it gives the framework's central state variable, *chao*, a dimensionless definition under which its threshold at unity is definitional rather than empirical. Third, it proves as theorems the two derivations that are genuinely theorems — the monotone decline of carrier reactivity with throughflow, and the divergence of adaptive tracking error under environmental acceleration — and states the assumptions on which each rests, together with the experiment that would break the load-bearing one.

1 Purpose and scope

The twelve derivations of the companion document are not all the same kind of object. Four are entailments: their conclusions follow from their premises, and denying one is a contradiction. Four are quantitative: they follow from an optimisation or a rate argument, conditional on a functional form that has not yet been written down. Four are abductive: they are the best available ordering of the evidence, and they are defeasible.

All twelve are currently written with the same arrow. That is the single largest unforced weakness in the presentation, because a reader encountering *universal* $\Rightarrow$ *ancestral* as a deductive claim will note that universality also admits convergence and lateral transfer, and will discount the entire set on the strength of one mislabelled step — despite the method section having already declared, correctly, that the framework proceeds by re-interpretation rather than by deduction.

This paper does not restate the framework, defend it, or supply its evidence. It supplies the machinery, so that the monograph can cite rather than carry it. Everything here is either a definition, a theorem with stated assumptions, or an explicit statement that something is *not* a theorem.

2 Inference typing

Definition 1 (Arrow types)**.** Let P be a premise set and Q a conclusion.

$P \vdash Q$ **Entailment.** Q follows from P under the stated definitions. Denying Q while asserting P is a contradiction. No empirical content is added by the step.

$P \rhd Q$ **Comparative statics or dynamics.** Q follows from P together with a stated functional form and a stated monotonicity or rate condition. The step is valid, but the functional form carries empirical content and is a place the chain can be attacked without attacking P.

$P \rightsquigarrow Q$ **Abduction.** Q is the ordering of P that leaves least unexplained. The step is defeasible; alternative Q' are admissible and the claim is comparative, not exclusive.

Entailment $\vdash$	**Quantitative** $\rhd$	**Abductive** $\rightsquigarrow$
D2 redundancy	D4 carrier fidelity	D1 gradient first
D3 all-axes filter	D8 canalisation	D5 two sweeps
D7 glycine freeze	D9 acceleration	D6 chirality date
D11 cycle closure	D10 saturation	D12 stress depth

Table 1: The twelve derivations by inference type. The abductive column is not a weaker column; it is the column doing the work the method claims to be doing. Marking it as such costs nothing and forecloses the obvious attack.

Remark 2. The typing is a property of the chain as written, not of the claim. D12 is abductive because its final step — from "the survivor's machinery is transition-tolerance machinery" to "transition-tolerance should be more deeply conserved" — is an empirical prediction about the conservation record, not an entailment. Its earlier steps are entailments. Where a chain changes type mid-way, each arrow should carry its own mark.

3 The state variable

3.1 What is required of it

The framework already uses *chao* as though it were a state variable. It has a first derivative (rate of environmental change), a second derivative (acceleration, which D9 makes load-bearing), a sign, a dimensionality, and a threshold at unity which the monograph corrects from an absolute constant to a basin-relative one: *one chao is a lip, not a constant.* What it does not have is units. Until it does, $d^2\chi/dt^2$ is a gesture, and D4, D8, D9 and D10 cannot be theorems however well they are argued.

The correction to basin-relativity is the constructive hint. A quantity that is relative to the basin the system currently occupies, and whose threshold sits at unity, is a ratio of a forcing to a barrier.

3.2 Definition

Let a system occupy a basin $\mathfrak{b}$ of its persistence landscape — in the framework's vocabulary, a canal. Write:

- $\Phi(t)$ — the free-energy flux density available to the system from its environment, units $\mathrm{J\,s^{-1}\,m^{-3}}$;

- $\Delta G_{\mathfrak{b}}$ — the escape barrier of $\mathfrak{b}$: the free energy per unit volume required to leave the basin, units $\mathrm{J\,m^{-3}}$;
- $\tau_{\mathfrak{b}}$ — the characteristic relaxation time of the system within $\mathfrak{b}$, units s.

Definition 3 (Chao).

$$\chi_{\mathfrak{b}}(t) \;\coloneqq\; \frac{\Phi(t)\,\tau_{\mathfrak{b}}}{\Delta G_{\mathfrak{b}}}.$$

$\Phi\tau_{\mathfrak{b}}$ has units $\mathrm{J\,m^{-3}}$, so $\chi_{\mathfrak{b}}$ is dimensionless. It is the ratio of the free energy the environment delivers within one relaxation time to the free energy required to leave the basin.

Corollary 4 (The lip is definitional)**.** $\chi_{\mathfrak{b}} = 1$ *exactly when the environment supplies, within one relaxation time, precisely the energy needed to overtop the basin. The threshold at unity is therefore a consequence of the definition and requires no empirical defence. Its basin-indexing is likewise definitional, since both* $\Delta G_{\mathfrak{b}}$ *and* $\tau_{\mathfrak{b}}$ *carry the index.*

This is a real gain. In the monograph the statement *one chao is a lip* is a substantive correction that a reader may contest. Under Definition 3 it is arithmetic, and the substantive content moves to where it belongs: the claim that Φ, ΔG and τ are the right three quantities.

3.3 Scale and dimension

The monograph's three regimes are recovered by choosing the basin.

- **Local >1 chao** — $\chi_{\mathfrak{b}} > 1$ for some $\mathfrak{b}$ at sub-planetary scale. A fire, a plague, an ice age.
- **Systemic >1 chao** — $\chi_{\mathfrak{b}} > 1$ for the basin containing the planetary system as a whole.
- **$\gg 1$ chao, Pandæmonium** — $\chi_{\mathfrak{b}} \gg 1$ simultaneously across a large fraction of the basin set. Every accessible phase change is explored.

Dimensionality becomes countable. Let Φ_i be the flux density along independent environmental axis i, and $\chi_i = \Phi_i\tau_{\mathfrak{b}}/\Delta G_{\mathfrak{b},i}$. Then

$$\dim \chi_{\mathfrak{b}} \;=\; \bigl|\{\, i : \chi_i > 1 \,\}\bigr|,$$

which formalises the monograph's claim that cold is the low-dimensional case: reducing temperature lowers Φ_i across most chemical axes while raising it on one, so the perturbation is high in magnitude and low in dimension. Magnitude and dimension are independent, which is why they must be reported separately.

Remark 5 (Relation to dissipative adaptation). Φ is the same quantity that appears as the driving term in dissipative adaptation. The numerator $\Phi\tau_{\mathfrak{b}}$ is bounded below by the work dissipated in any trajectory that leaves $\mathfrak{b}$ within $\tau_{\mathfrak{b}}$, so Definition 3 connects directly to the Crooks and Jarzynski relations. An alternative numerator built from the Kullback–Leibler divergence of the system's distribution from equilibrium is available and sits closer to the stochastic-thermodynamics literature; it is not used here because the flux formulation keeps Φ directly measurable from the geochemical record, which is what the inversion method needs.

4 Canalisation and the threshold (D8)

D8 as written in the companion document says: *canalisation deepens a channel; the lip is the threshold; canalisation depth is threshold height; the same perturbation overtops a shallow channel and not a deep one* — landing on the selective extinction of specialists.

Written that way the polarity is left to the reader. Specialists are the deep channels, and deep channels are the ones that are *not* overtopped, so "overtopping" cannot mean death. It means escape. The resolution requires a second quantity, and stating it is a genuine gain from formalisation rather than a repair.

Definition 6 (Basin viability)**.** Let $V_{\mathfrak{b}}(t) \in [0, 1]$ denote the degree to which throughflow within $\mathfrak{b}$ still supports the system's dissipative structure. $V_{\mathfrak{b}} \to 0$ means the basin no longer sustains persistence, independently of whether the system can leave it.

Proposition 7 (Specialists are trapped, generalists are liberated)**.** *Let canalisation depth $\Delta G_{\mathfrak{b}}$ be increasing in degree of specialisation. Under a perturbation that raises Φ uniformly across basins while degrading $V_{\mathfrak{b}}$:*

(a) lineages with $\Delta G_{\mathfrak{b}} < \Phi\tau_{\mathfrak{b}}$ have $\chi_{\mathfrak{b}} > 1$ and leave their basin — exploration;

(b) lineages with $\Delta G_{\mathfrak{b}} > \Phi\tau_{\mathfrak{b}}$ have $\chi_{\mathfrak{b}} < 1$, cannot leave, and persist only while $V_{\mathfrak{b}} > 0$.

Hence generalists are liberated and specialists are extinguished in place, by the same perturbation, with no additional assumption.

Proof. Immediate from Definition 3: $\chi_{\mathfrak{b}} \gtrless 1$ according as $\Phi\tau_{\mathfrak{b}} \gtrless \Delta G_{\mathfrak{b}}$. The monotonicity of $\Delta G_{\mathfrak{b}}$ in specialisation partitions lineages between the two cases. Extinction in case (b) follows from $V_{\mathfrak{b}} \to 0$ with escape unavailable. $\square$

The proposition also predicts the intermediate case, which the prose version does not: a lineage with $\Delta G_{\mathfrak{b}} \approx \Phi\tau_{\mathfrak{b}}$ escapes into an adjacent basin rather than either exploring freely or dying, and should appear in the record as a rapid narrow radiation rather than as either a mass extinction victim or a broad adaptive radiation.

5 Carrier reactivity falls with throughflow (D4)

This is the framework's strongest formal result and is currently presented as prose. It is a monotone comparative statics problem and it has a clean proof.

5.1 Setup

Let $r > 0$ index the reactivity of the heritable carrier — the rate constant of the chemistry that both builds it and destroys it. Let $J \geq 0$ denote throughflow: the rate at which free energy and material are supplied to the system.

Define $W(r, J)$ as the rate at which the population accumulates *retained* adaptive information, decomposed as

$$W(r, J) \;=\; s(r, J) \;-\; \ell(r),$$

where s is the rate of productive synthesis — new variants generated and expressed — and ℓ is the rate of information loss through spontaneous degradation and miscopying.

Assumption A1 (Monotonicity)**.** s is nondecreasing in r and in J.

Assumption A2 (Complementarity)**.** s is supermodular in (r, J): for $r' \geq r$ and $J' \geq J$,

$$s(r', J') - s(r, J') \;\geq\; s(r', J) - s(r, J).$$

Where s is twice differentiable this is $\partial^2 s/\partial r\, \partial J \geq 0$.

Assumption A3 (Loss is autonomous)**.** ℓ is increasing in r and does not depend on J.

Assumption A2 is the one that carries the result, and it is an empirical claim about chemistry rather than a mathematical convenience. Its content: a more reactive carrier is worth more when supply is abundant than when supply is scarce, because under supply limitation throughput is set by J and raising the rate constant buys nothing. Under supply excess throughput is set by the rate constant and raising it buys proportionally. Assumption A3 says degradation is driven by the carrier's own chemistry — hydrolysis, oxidation, intramolecular attack — and proceeds whether or not the environment is supplying anything.

5.2 The theorem

Theorem 8 (Optimal carrier reactivity is nondecreasing in throughflow)**.** *Under Assumptions A1–A3, the optimal reactivity*

$$r^*(J) \;\in\; \operatorname*{arg\,max}_{r \in R} \; W(r, J)$$

is nondecreasing in J in the strong set order, on any lattice $R \subseteq \mathbb{R}_{>0}$.

Proof. $W(r, J) = s(r, J) - \ell(r)$. The subtracted term depends on r alone (Assumption A3), so for $r' \geq r$ and $J' \geq J$,

$$\begin{aligned} &\big[W(r', J') - W(r, J')\big] - \big[W(r', J) - W(r, J)\big] \\ &\quad = \big[s(r', J') - s(r, J')\big] - \big[s(r', J) - s(r, J)\big] \;\geq\; 0 \end{aligned}$$

by Assumption A2, the ℓ terms cancelling exactly. Hence W is supermodular in (r, J) on $R \times \mathbb{R}_{\geq 0}$, which is a lattice. Topkis's monotonicity theorem gives that $\arg\max_r W(r, J)$ is nondecreasing in J in the strong set order. □

Remark 9. Neither concavity nor differentiability nor uniqueness of the maximiser is required. This matters: the fitness landscape over carrier chemistries is neither smooth nor single-peaked, and any proof requiring first-order conditions would be assuming away the structure of the problem. Supermodularity is the weakest condition that delivers monotone comparative statics, and Milgrom–Shannon single crossing weakens it further if Assumption A2 proves too strong: single crossing of $W(r', \cdot) - W(r, \cdot)$ suffices for the same conclusion.

Corollary 10 (The carrier sequence)**.** *Let throughflow J be nondecreasing in χ. Then $r^*(\chi)$ is nondecreasing in χ, and since planetary chao falls monotonically over the interval of interest, optimal carrier reactivity falls monotonically. The observed order*

$$\textit{sugar} \;\rightarrow\; \textit{RNA} \;\rightarrow\; \textit{RNA–protein} \;\rightarrow\; \textit{DNA}$$

is monotone decreasing in r and is therefore the predicted order, not an observation requiring separate explanation.

Corollary 11 (The 2′-OH)**.** *DNA is RNA with the 2′-hydroxyl removed. The 2′-OH is the intramolecular nucleophile responsible for backbone cleavage: removing it lowers ℓ and lowers r simultaneously. It is therefore a movement along the r axis in the direction Theorem 8 predicts, and DNA is not a better carrier than RNA. It is a better carrier* later.

5.3 How to break it

Assumption A2 is falsifiable and the experiment is cheap. In a chemostat, vary the dilution rate to move between supply-limited and supply-excess regimes, and measure the marginal fitness return to catalytic rate constant in each. Supermodularity predicts the marginal return is strictly greater under supply excess. If the marginal return is flat or inverted — if extra reactivity pays the same or better under limitation — Theorem 8 fails and Corollary 10 loses its derivation, which would leave the carrier sequence as an observation in need of a different mechanism.

That is the honest local falsifier for D4, and it is a bench experiment rather than a geological argument.

6 Punctuation tracks acceleration (D9)

D9 asserts that adaptation has a response time, that steady change is trackable at any speed, that accelerating change is not, and that $d^2\chi/dt^2$ therefore decides who passes the filter. This is a tracking problem and the result is exact.

6.1 Setup

Let $\theta(t)$ be the environmental optimum — the phenotypic state that maximises persistence under current conditions — and let $a(t)$ be the population's realised adapted state. Model adaptation as a first-order lag with time constant τ:

$$\dot{a} \;=\; \frac{\theta(t) - a(t)}{\tau}. \tag{1}$$

Write the tracking error $e(t) = \theta(t) - a(t)$, so that

$$\dot{e} \;+\; \frac{e}{\tau} \;=\; \dot{\theta}. \tag{2}$$

Let e_{crit} be the lineage's tolerance: persistence fails when $|e| > e_{\text{crit}}$.

Assumption A4 (Forcing transfer)**.** θ is a monotone, boundedly differentiable function of χ, so that $\ddot{\theta} = 0$ if and only if $\ddot{\chi} = 0$ up to a bounded factor.

6.2 The theorem

Theorem 12 (Magnitude is survivable; acceleration is not)**.** *Let e evolve by* (2) *with* $\tau > 0$.

(a) If $\ddot{\theta} = 0$, so that $\dot{\theta} = v$ is constant, then $e(t) \to \tau v$ and $|e(t)| \le \max\{|e(0)|, \tau|v|\}$ for all t. A lineage with $e_{\text{crit}} > \tau|v|$ persists indefinitely, for arbitrarily large $|v|$.

(b) *If $\ddot{\theta} = c \neq 0$, then $e(t) = \tau c\, t - \tau^2 c + O(e^{-t/\tau})$, which is unbounded. Every lineage fails at finite time*

$$t_{\text{ext}} \;=\; \frac{e_{\text{crit}}}{\tau|c|} \;+\; \tau \;+\; O(e^{-t/\tau}),$$

regardless of e_{crit}.

Proof. (a) With $\dot{\theta} = v$, (2) is linear with constant forcing; the particular solution is $e_p = \tau v$ and the homogeneous solution decays as $e^{-t/\tau}$. Hence $e(t) = \tau v + (e(0) - \tau v)e^{-t/\tau}$, which is monotone between $e(0)$ and τv and therefore bounded by their maximum in absolute value.

(b) With $\dot{\theta} = ct$, try $e_p = \alpha t + \beta$. Substituting into (2): $\alpha + (\alpha t + \beta)/\tau = ct$, giving $\alpha = \tau c$ and $\beta = -\tau\alpha = -\tau^2 c$. The homogeneous part decays. So $e(t) = \tau ct - \tau^2 c + O(e^{-t/\tau})$, linear and unbounded. Setting $|e(t_{\text{ext}})| = e_{\text{crit}}$ and solving gives the stated time. ☐

Corollary 13 (Mechanism for punctuation)**.** *Under Assumption A4, extinction intensity is set by $\ddot{\chi}$ and not by $\dot{\chi}$. A steadily changing environment, however fast, admits a stable distribution of surviving tolerances; an accelerating one admits none. Punctuation in the fossil record is therefore the signature of $\ddot{\chi} \neq 0$, and interludes of stasis are intervals of $\ddot{\chi} \approx 0$ — which may include intervals of rapid but steady change.*

Remark 14 (Discriminating D9 from magnitude accounts)*.* Part (a) is the discriminating half. A magnitude account predicts extinction intensity scales with $|\dot{\chi}|$; Theorem 12 predicts intervals of large $|\dot{\chi}|$ with $\ddot{\chi} \approx 0$ show *no* elevated extinction. Those intervals exist in the record and the two accounts differ sharply across them. This is a stratigraphic test with a clean null.

Remark 15 (Where the model is thin)*.* (1) is a first-order lag with a single τ. Real adaptive response is distributed across traits with different time constants, which converts the scalar τ into a spectrum and e_{crit} into a trait-weighted norm. Part (b) survives this generalisation — a linear system of any order still fails to track a parabolic input with bounded error — but part (a)'s bound becomes $\|\tau\|\,|v|$ for an appropriate operator norm, and the crisp statement "arbitrarily large $|v|$ is survivable" weakens to "any $|v|$ below a lineage-specific ceiling". The qualitative asymmetry between magnitude and acceleration is unaffected.

7 Two shorter results

7.1 Cycle closure requires retention (D11)

Proposition 16. *Let a reaction network be represented as a hypergraph on species S, and let species $x \in S$ have mean residence time λ_x. Let σ be a directed cycle in the network with transit time T_σ — the time from production of a species to its consumption in closing the cycle. If $\lambda_x < T_\sigma$ for all $x \in \sigma$, then σ does not close, and the network supports no autocatalytic cycle.*

Proof. Closure of σ requires that the species produced at each step persist until consumed at the next. If every $\lambda_x < T_\sigma$ the expected occupancy of each intermediate at its consumption time is zero to leading order, so no closed flux is sustained. ☐

Corollary 17 (Venus, and a mission discriminator)**.** *A system generating chemical disequilibrium without retention supports only acyclic, throughflow-driven signatures. Its observable atmospheric signature is therefore unpaired: species produced without their* *cycle-closing partners. A biological system, having retention, produces paired signatures. The discriminator is the pairing, not the disequilibrium.*

7.2 Separability and the virus (definitional)

Definition 18 (Life, decomposed)**.** For a system x, write

$$\mathrm{Life}(x) \;\equiv\; D(x) \wedge H(x) \wedge F(x),$$

where $D(x)$: x is a dissipative structure held above threshold by sustained throughflow; $H(x)$: x carries a heritable carrier; $F(x)$: that carrier is subject to persistence filtering. Let $C(x, y) \in [0, 1]$ denote the degree to which x's carrier is coupled to y's dissipative structure, with $C = 0$ uncoupled and $C = 1$ permanently integrated.

Proposition 19 (Separability yields a chain where conjunction yields a point)**.** *Under the conjunctive reading, the free virion, the lytic infection, the lysogenic prophage and the endogenous retrovirus all satisfy* $\neg\,\mathrm{Life}$ *and are indistinguishable. Under the decomposed reading with graded C, they satisfy $H \wedge \neg D$ throughout and are totally ordered by C:*

$$C(\mathit{virion}) = 0 \;<\; C(\mathit{lytic}) \;<\; C(\mathit{lysogen}) \;<\; C(\mathit{ERV}) = 1.$$

The proposition is trivial mathematically and load-bearing rhetorically: it is the formal content of the claim that the virus is not a case the definition struggles with but a case the definition resolves. The syncytin question — at what point the viral carrier stopped being a passenger — is answered by the ordering, at $C = 1$.

Remark 20. C is the same object as D2's conversion category. Lysogeny is the mechanism by which an exogenous source of redundancy becomes an endogenous one, so the coupling gradient and the redundancy routes are one structure described twice. That the two descriptions coincide was not built in, and is a consistency check the framework passes.

8 What is proved and what is not

Result	**Status**	**Rests on**
Chao (Def. 3)	Definition	Choice of Φ, ΔG, τ as the right three quantities
Threshold at unity	Corollary of definition	Nothing further
Prop. 7 (D8)	Proved	$\Delta G_{\mathfrak{b}}$ increasing in specialisation
Thm. 8 (D4)	Proved	AA1–AA3; AA2 load-bearing
Cor. 10	Proved	Thm. 8 plus J nondecreasing in χ
Thm. 12 (D9)	Proved	First-order lag; AA4
Prop. 16 (D11)	Proved	Residence times below transit time
Virus chain	Definitional	Nothing further
D1, D5, D6, D12	*Not proved*	Abductive; see typing table
D2, D3, D7	Entailments	Not restated here
D10	*Not proved*	Awaits a saturation model

Table 2: Status of every claim touched by this paper. Four of the twelve derivations remain abductive and are marked as such; that is a statement about the notation, not a retreat from the claims.

8.1 The open one

D10 — *nodes saturate, edges proliferate* — is quantitative in type but has no model here. The shape of the missing argument is clear: component space is bounded, so exploration of components saturates on a curve of the occupancy-saturation family, while relational space grows combinatorially in the number of components, so the marginal novelty available per unit exploration crosses over from components to relations at a computable point. Writing that crossover down would convert D10 from $\rhd$ to a theorem, and would put a date on it. It is the most tractable open item in this document.

8.2 Global falsifier, restated

The companion document's global falsifier stands and is unaffected by anything here: if the derivations systematically land on observations that already have accepted mechanistic explanations, the method is not doing work and the framework reduces to a vocabulary. That test is prior to this paper. Formalisation raises the cost of dismissing individual derivations; it does not touch the question of whether the set of landings is real.

9 Note on venue and relation to the monograph

This paper is separable from *The First Filter* and should precede it. It establishes χ, proves the two results the monograph currently carries as prose, and marks the inference types — after which the monograph can cite rather than carry the machinery, which is a considerably stronger position than the alternative.

Appropriate venues are *Journal of Theoretical Biology*, *Journal of Mathematical Biology*, or *Bulletin of Mathematical Biology*; *Interface Focus* if the origins framing is to be foregrounded. Preprint to arXiv `q-bio.PE` with cross-list to `cond-mat.stat-mech`, which is where the dissipative-adaptation readership sits.

Draft v0.1. Companion to *Claims and Derivations* v1.0 and *The First Filter* Volume I. Nothing in this document reports executed results.